

Dissipation and decoherence induced by collective dephasing in coupled-qubit system with a common bath

Z. H. Wang,¹ Y. J. Ji,¹ Yong Li,^{1,2} and D. L. Zhou³

¹*Beijing Computational Science Research Center, Beijing 100084, China*

²*Synergetic Innovation Center of Quantum Information and Quantum Physics,
University of Science and Technology of China, Hefei 230026, China*

³*Beijing National Laboratory for Condensed Matter Physics,
Institute of Physics, Chinese Academy of Sciences, Beijing 100190, China*

The longitudinal coupling of a system to the bath usually induces the pure dephasing of the system. In this paper, we study the collective dephasing induced dissipation and decoherence in a coupled-qubit system with a common bath. It is shown that, compared with the case of the same system with independent baths, the interference between the dephasing processes of different qubits induced by the common bath significantly changes the dissipation of the system. For the system of two coupled qubits, the interference leads to a faster decoherence in the non-single-excitation subspaces and a slower dissipation (and decoherence) in the single-excitation subspace. For the system of multiple coupled qubits, we also find the slower dissipation in the single-excitation subspace and obtain the decay rates of the first excited states for different system sizes numerically. All our results on collective dephasing induced dissipation can be explained based on a simple model with Fermi's golden rule.

PACS numbers: 03.65.Nk, 03.67.Lx, 78.67.-n

I. INTRODUCTION

The dynamics of quantum open systems [1] has attracted more and more attentions in the fields of quantum optics and quantum information [2, 3]. The influence of the bath on a single qubit is characterized by two times: the relaxation time T_1 and the decoherence time T_2 satisfying $1/T_2 = 1/2T_1 + 1/T_\varphi$, where T_1 is determined by the transverse coupling to the bath with the frequency near resonance with that of the qubit while T_φ , named as “pure dephasing time” [4, 5], arises from the longitudinal coupling to the bath with much lower frequencies.

In the system of superconducting qubits [6], the main intrinsic element that limits the decoherence time results from the low-frequency noise [7–10]. Similarly, in some biology systems such as the excitations in pigment-protein complexes [11, 12], the phonon modes in the bath have much lower frequencies than the transition frequency of the excitations. In such kinds of systems, how to efficiently suppress the decoherence arising from the low frequency noise is a central task.

Besides the single system embedded in the bath, the emergent properties of many-body open system are also at the frontier of many research fields. Due to the cooperative effect in coupled system with subsystems interacting with a correlated [13, 14] or common [15–22] bath, it usually exhibits some exotic collective phenomena. However, in most of the above literatures, the authors mainly discussed the entanglement induced by many-body interaction and the coupling to the bath, instead of the interference effect arising from the common or correlated bath(s). Actually, the interference effect in atomic ensemble leads to the well-known “single-photon superradiance” [23] in which the decay rate of the single-excitation state is proportional to the number of the atoms involved

due to the collective effect. A natural question is what will the collective dephasing behave in the coupled systems when the subsystems share a common bath?

In this paper, we study the collective dephasing of multiple coupled qubits sharing a common bath. Due to the coupling between the neighbour qubits and the interference effect induced by the common bath, the collective dephasing leads to the dissipation of the excited states and decoherence from the viewpoint of energy representation of the system. Based on our previous work about the interference effect in multi-channel dissipation of the system [19], we continue to investigate the dynamics of two coupled-qubit with a common bath in detail. The system under consideration has an excitation conservation character and shows dramatic difference in the non-single-excitation and single-excitation subspaces. In the non-single-excitation subspaces, the dephasing of the two qubits interferes with each other constructively, leading to a faster decoherence but without dissipation. On the contrary, the dephasing of the qubits interferes destructively in the single-excitation subspace, and it shows slower dissipation and decoherence processes. To demonstrate the underlying mechanism, we reformulate the master equation to include the interference effect of the bath, and compare the signs between the self-dephasing terms and their interference terms to explain the faster and slower decay of the system. Moreover, we map the system of two coupled qubits into a single-qubit model. It is mapped into a spin-boson model [24] in the single-excitation subspace and into a pure dephasing model in the non-single-excitation subspaces. Then, the dynamics of the system can be obtained intuitively without detailed calculation.

Furthermore, we extend our discussions to the system of multiple coupled qubits. Similar to the two-qubit sys-

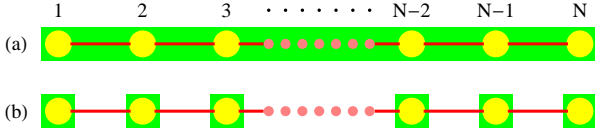


FIG. 1. (Color online) The schematic diagram of the N coupled identical qubits interacting longitudinally with a common bath (a) and with their independent baths (b). Here the qubits, the interaction between neighbor qubits, and the baths are represented by yellow balls, red links, and green rectangles respectively.

tem, we find that the decay rates of the eigen-states are much smaller when the qubits share a common bath than the situation with independent baths. We obtain the decay rates of the first excited states in the single-excitation subspace by Fermi's golden rule [25, 26] for three-qubit system and by numerical calculation for the system of more qubits.

The rest of the paper is organized as follows. In Sec. II, we present our model as multiple coupled qubits interacting longitudinally with the common or independent bath(s). In Sec. III, we investigate the collective dephasing induced dissipation and decoherence for two coupled qubits sharing a common bath in detail, and extend the discussions to the system composed of multiple coupled qubits in Sec. IV. In Sec. V, we give some remarks and draw the conclusion.

II. THE MODEL

As shown in Fig. 1, the system under consideration is composed of an align of N identical coupled qubits, labelled by the index $1, 2, \dots, N$, respectively, interacting with their common [Fig. 1(a)] or independent [Fig. 1(b)] bath(s).

The Hamiltonian of the system is

$$H_S = \frac{\Omega}{2} \sum_{i=1}^N \sigma_z^{(i)} + \lambda \sum_{i=1}^{N-1} (\sigma_-^{(i)} \sigma_+^{(i+1)} + h.c.), \quad (1)$$

where Ω is the energy level spacing between the ground state $|g\rangle$ and excited state $|e\rangle$ of each qubit. σ_z , σ_- and σ_+ are the traditional Pauli operators for two-level system. λ is the coupling strength between arbitrary two neighbor qubits.

Since any quantum system in nature cannot be absolutely isolated from its surrounding bath, we must regard the quantum systems as open systems [1]. For a quantum system, when the frequency of the mode in the bath is comparable to the characteristic frequency of the system, it will induce the dissipation and decoherence simultaneously. However, when the frequency of the bath is much lower than that of the system, it contributes only to the decoherence, which is named as pure dephasing. In the rest of the paper, we will study the collective behavior

of multiple coupled-qubit system in such kinds of pure dephasing processes.

Firstly, we consider the situation where all the qubits share a common bath as shown in Fig. 1(a). The Hamiltonian of the whole system then reads $H^{\text{com}} = H_S + H_B^{\text{com}} + H_I^{\text{com}}$, where $H_B^{\text{com}} = \sum_j \omega_j b_j^\dagger b_j$ describes the free terms of the bosonic modes in the bath. Here, b_j is the annihilation operator of the j th mode with frequency ω_j . In our consideration, it satisfies $\omega_j \ll \Omega$ for any j . The interaction between the system and the bath is described by

$$H_I^{\text{com}} = \sum_i \sum_j \kappa_{i,j} \sigma_z^{(i)} (b_j^\dagger + b_j), \quad (2)$$

where $\kappa_{i,j}$ is the coupling strength between the i th qubit and the j th mode in the bath.

Secondly, for the case that each qubit interacts with its local bath independently as shown in Fig. 1(b), the Hamiltonian is written as $H^{\text{ind}} = H_S + H_B^{\text{ind}} + H_I^{\text{ind}}$, with $H_B^{\text{ind}} = \sum_{i,j} \omega_{ij} c_{ij}^\dagger c_{ij}$ where c_{ij} is the annihilation operator of the j th mode in the bath for the i th qubit and ω_{ij} ($\ll \Omega$) is its frequency. The bath-system interaction is

$$H_I^{\text{ind}} = \sum_i \sum_j \kappa_{i,j} \sigma_z^{(i)} (c_{ij}^\dagger + c_{ij}). \quad (3)$$

The complete information about the effect of the bath and the coupling with the system is encapsulated in the spectral function $J_i(\omega)$ which is defined by the expression [24]

$$J_i(\omega) \equiv \pi \sum_j \kappa_{i,j}^2 \delta(\omega - \omega_j) \quad (4)$$

for both the independent and common bath.

In what follows, we will consider the ohmic dissipation with the spectrum functions being expressed as

$$J_i(\omega) = \chi_i \omega \coth(\omega), \quad (5)$$

where χ_i is the dissipation coefficient.

III. DYNAMICS IN TWO COUPLED-QUBIT SYSTEM

In this section, we will study the collective dephasing process of two coupled qubits sharing a common bath, and analyze the quantum interference effect in detail. As the central result of this paper, we will show that the collective dephasing of the qubits leads to the dissipation and decoherence of the system in the energy representation.

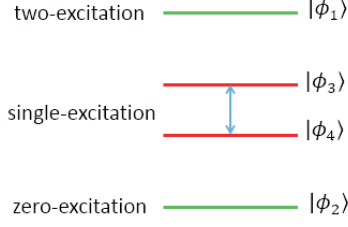


FIG. 2. (Color online) The energy spectrum of H_S . The green lines represent the eigen-states in the non-single-excitation (i.e. two- and zero-excitation) subspaces while the red lines represent the eigen-states in the single-excitation subspace. The blue arrow implies the transition induced by H_I^{com} .

A. Master equation

For the system of two coupled qubits, the Hamiltonian H_S is reduced to

$$H_S = \frac{\Omega}{2}(\sigma_z^{(1)} + \sigma_z^{(2)}) + \lambda(\sigma_-^{(1)}\sigma_+^{(2)} + h.c.). \quad (6)$$

Correspondingly, the interaction between the qubits and their common bath is written as

$$H_I^{\text{com}} = \sum_j (\kappa_{1j}\sigma_z^{(1)} + \kappa_{2j}\sigma_z^{(2)})(b_j^\dagger + b_j). \quad (7)$$

We first diagonalize the Hamiltonian H_S , whose eigenstates are

$$|\phi_1\rangle = |e; e\rangle, \quad (8a)$$

$$|\phi_2\rangle = |g; g\rangle, \quad (8b)$$

$$|\phi_3\rangle = \frac{1}{\sqrt{2}}(|e; g\rangle + |g; e\rangle), \quad (8c)$$

$$|\phi_4\rangle = \frac{1}{\sqrt{2}}(-|e; g\rangle + |g; e\rangle), \quad (8d)$$

and the corresponding eigen-energies E_i are

$$E_1 = -E_2 = \Omega, \quad (9a)$$

$$E_3 = -E_4 = \lambda, \quad (9b)$$

respectively.

In our system, the total excitations of the qubits are conserved with $|\phi_3\rangle$ and $|\phi_4\rangle$ being in the single-excitation subspace, while $|\phi_1\rangle$ and $|\phi_2\rangle$ being in the subspaces with two and zero excitations respectively, which are non-single-excitation subspaces. It is apparent that the interaction between the qubits and the bath couples $|\phi_3\rangle$ and $|\phi_4\rangle$, but not $|\phi_1\rangle$ and $|\phi_2\rangle$ as shown in Fig. 2, where we illustrate the energy spectrum of H_S .

Following the standard steps to derive the master equation [27], we obtain the master equation in the eigenrepresentation of H_S as

$$\frac{d\rho_{cd}}{dt} = -i\omega_{cd}\rho_{cd} + \sum_{k,l} \gamma^{cdkl} \rho_{kl}. \quad (10)$$

Here, $\rho_{cd} = \langle c|\rho|d\rangle$ ($c, d = \phi_i$, for $i = 1, 2, 3, 4$) is the element of the reduced density matrix of the coupled-qubit system, and $\omega_{cd} = E_c - E_d$ is the energy difference between the states $|c\rangle$ and $|d\rangle$. In Eq. (10), $\gamma^{cdkl} = \sum_{i=1}^4 \gamma_i$ with

$$\begin{aligned} \gamma_1 &= -\delta_{dl} \sum_n \theta(\omega_{kn}) [J_1(\omega_{kn}) Z_{cn}^{(1)} Z_{nk}^{(1)} + J_2(\omega_{kn}) Z_{cn}^{(2)} Z_{nk}^{(2)} \\ &\quad + \sqrt{J_1(\omega_{kn}) J_2(\omega_{kn})} (Z_{cn}^{(2)} Z_{nk}^{(1)} + Z_{cn}^{(1)} Z_{nk}^{(2)})], \\ \gamma_2 &= \theta(\omega_{kc}) [J_1(\omega_{kc}) Z_{ck}^{(1)} Z_{ld}^{(1)} + J_2(\omega_{kc}) Z_{ck}^{(2)} Z_{ld}^{(2)} \\ &\quad + \sqrt{J_1(\omega_{kc}) J_2(\omega_{kc})} (Z_{ck}^{(2)} Z_{ld}^{(1)} + Z_{ck}^{(1)} Z_{ld}^{(2)})], \\ \gamma_3 &= -\delta_{ck} \sum_n \theta(\omega_{ln}) [J_1(\omega_{ln}) Z_{ln}^{(1)} Z_{nd}^{(1)} + J_2(\omega_{ln}) Z_{ln}^{(2)} Z_{nd}^{(2)} \\ &\quad + \sqrt{J_1(\omega_{ln}) J_2(\omega_{ln})} (Z_{ln}^{(2)} Z_{nd}^{(1)} + Z_{ln}^{(1)} Z_{nd}^{(2)})], \\ \gamma_4 &= \theta(\omega_{ld}) [J_1(\omega_{ld}) Z_{ck}^{(1)} Z_{ld}^{(1)} + J_1(\omega_{ld}) Z_{ck}^{(2)} Z_{ld}^{(2)} \\ &\quad + \sqrt{J_1(\omega_{ld}) J_2(\omega_{ld})} (Z_{ck}^{(2)} Z_{ld}^{(1)} + Z_{ck}^{(1)} Z_{ld}^{(2)})], \end{aligned} \quad (11)$$

where we have shorten σ_z as Z for the sack of compactness and $A_{cd} = \langle c|A|d\rangle$ is the matrix element of the operator A in the energy representation of H_S . δ_{dl} and δ_{ck} are the usually used Dirac δ functions. The function $\theta(x)$ is defined as

$$\theta(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}. \quad (12)$$

In Eq. (11), the terms proportional to $\sqrt{J_1(\cdot)J_2(\cdot)}$ represent the contribution of the quantum interference between the two dephasing channels.

B. Collective dephasing induced dissipation

Firstly, we confine ourselves in the non-single-excitation subspaces, then the off-diagonal elements of the density matrix satisfy

$$\frac{d}{dt}\rho_{12} = -2i\Omega\rho_{12} - \Gamma_{12}\rho_{12}, \quad (13)$$

whose solution is given by

$$\rho_{12}(t) = \rho_{12}(0) \exp[(-2i\Omega - \Gamma_{12})t]. \quad (14)$$

Here the decay rate of ρ_{12} is

$$\begin{aligned} \Gamma_{12} &= 4[\sqrt{J_1(0)} + \sqrt{J_2(0)}]^2 \\ &= 4[J_1(0) + J_2(0) + 2\sqrt{J_1(0)J_2(0)}]. \end{aligned} \quad (15)$$

Besides, the diagonal elements of the density matrix satisfy $d\rho_{11}/dt = d\rho_{22}/dt = 0$, that is, ρ_{11} and ρ_{22} do not decay during the time evolution.

It shows in Eq. (14) that the dephasing of the two qubits actually leads to the decoherence of the system in the energy representation. Attentions should be

paid that the last term in the second line of Eq. (15) comes from the quantum interference effect between the two dephasing channels arising from the common bath. When the two qubits interact with their local baths, this term will disappear, and the decay rate becomes $\Gamma'_{12} = 4[J_1(0) + J_2(0)]$, which is smaller than Γ_{12} . Therefore, the common bath enhances the decoherence of the system in the energy representation.

The mechanism underlying the faster decoherence can be observed in Eq. (11). Let us explain the quantum interference effect from the expressions of γ_i . In the non-single-excitation subspaces, the terms resulted from the interference of the dephasing of the two qubits have the same sign with their self-dephasing term, that is

$$Z_{ii}^{(1)} Z_{ii}^{(2)} = Z_{ii}^{(2)} Z_{ii}^{(1)} = Z_{ii}^{(1)} Z_{ii}^{(1)} = Z_{ii}^{(2)} Z_{ii}^{(2)} = 1 \quad (16)$$

for $i = 1, 2$. Therefore, the dephasing processes of two qubits constructively interfere with each other, leading to a faster decoherence.

Next, let us move to the single-excitation subspace. Then the elements of the reduced density matrix satisfy

$$\frac{d}{dt}\rho_{33} = -2\Gamma_{33}\rho_{33}, \quad (17)$$

$$\frac{d}{dt}\rho_{34} = -2i\lambda\rho_{34} + \Gamma_{34}(\rho_{43} - \rho_{34}), \quad (18)$$

where the decay rate Γ_{33} and decoherence rate Γ_{34} are

$$\begin{aligned} \Gamma_{33} = \Gamma_{34} &= [\sqrt{J_1(\omega_{34})} - \sqrt{J_2(\omega_{34})}]^2 \\ &= J_1(\omega_{34}) + J_2(\omega_{34}) - 2\sqrt{J_1(\omega_{34})J_2(\omega_{34})}. \end{aligned} \quad (19)$$

The last term in Eq. (19), which comes from the interference of dephasing between the two qubits, slows down the dissipation and decoherence of the whole system. Especially, when the spectra satisfy $J_1(\omega_{34}) = J_2(\omega_{34})$, the single-excitation subspace is a decoherence free subspace [28–30], in which there is neither dissipation nor decoherence.

To clearly show the dissipation and decoherence of the system in the single-excitation subspace, we define the variables

$$P(t) := \frac{|\rho_{33}(t)|}{|\rho_{33}(0)|}, \quad Q(t) := \frac{|\rho_{34}(t)|}{|\rho_{34}(0)|}, \quad (20)$$

and plot the time evolution of $P(t)$ and $Q(t)$ in Fig. 3 with the assumption that the system is initially prepared in the state $|e;g\rangle$. For comparison, we consider both the cases in which the two coupled qubits interact with a common (blue solid line) and two independent baths (red dashed line). It is shown in Fig. 3 that, when the two qubits share a common bath, the decays of ρ_{33} and ρ_{34} are much slower than the case with independent bathes due to the instructive interference. The instructive interference comes from the fact that

$$Z_{ij}^{(1)} Z_{ij}^{(2)} = Z_{ij}^{(2)} Z_{ij}^{(1)} = -Z_{ij}^{(1)} Z_{ij}^{(1)} = -Z_{ij}^{(2)} Z_{ij}^{(2)} = -1 \quad (21)$$

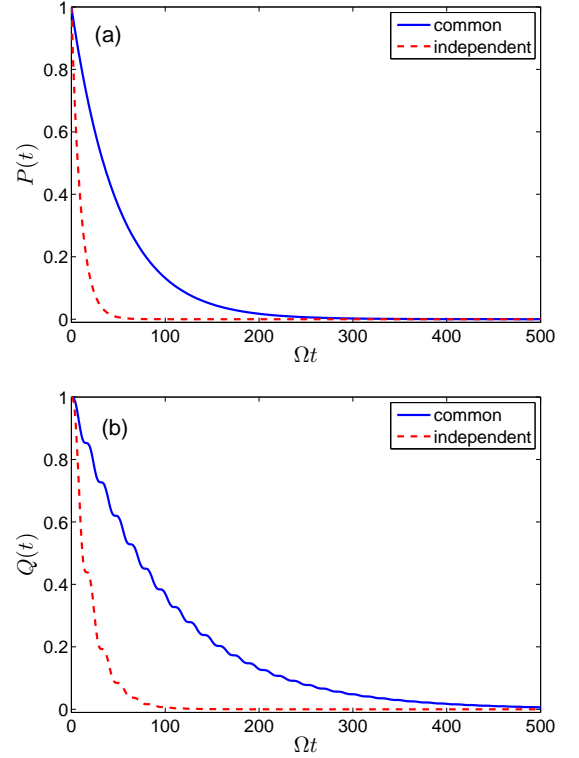


FIG. 3. (Color online) The time evolution of (a) $P(t)$ and (b) $Q(t)$. The parameters are set as $\lambda = 0.1$, $\chi_1 = 0.04$, and $\chi_2 = 0.01$ in units of $\Omega = 1$.

for $i, j = 3, 4$. It is just opposite with Eq. (16) that the interference term $Z_{ij}^{(1)} Z_{ij}^{(2)}$ and $Z_{ij}^{(2)} Z_{ij}^{(1)}$ have opposite signs with the self-dephasing terms $Z_{ij}^{(1)} Z_{ij}^{(1)}$ and $Z_{ij}^{(2)} Z_{ij}^{(2)}$. Therefore, the interference effect leads to a slower dissipation and decoherence in the single-excitation subspace.

C. Mapping to a single qubit

For the situation of two coupled qubits sharing a common bath which couples to the qubits longitudinally, it can be mapped to the model of a single qubit exposing in the bath.

For the system of two coupled qubits, the interaction between the qubits and the common bath is written as

$$\begin{aligned} H_I^{\text{com}} &= \sum_j \frac{\kappa_{1j} + \kappa_{2j}}{2} (\sigma_z^{(1)} + \sigma_z^{(2)}) (b_j + b_j^\dagger) \\ &\quad + \sum_j \frac{\kappa_{1j} - \kappa_{2j}}{2} (\sigma_z^{(1)} - \sigma_z^{(2)}) (b_j + b_j^\dagger). \end{aligned} \quad (22)$$

In the single-excitation subspace, it is obvious that $\sigma_z^{(1)} + \sigma_z^{(2)} = 0$, then the corresponding interaction Hamil-

tonian becomes

$$\begin{aligned} H_{I,s}^{\text{com}} &= \sum_j \frac{\kappa_{1j} - \kappa_{2j}}{2} (\sigma_z^{(1)} - \sigma_z^{(2)}) (b_j + b_j^\dagger) \\ &= -(|\phi_3\rangle\langle\phi_4| + |\phi_4\rangle\langle\phi_3|) \sum_j (\kappa_{1j} - \kappa_{2j}) (b_j + b_j^\dagger), \end{aligned} \quad (23)$$

where the second subscribe “s” implies the single-excitation subspace. We further define the collective operators

$$J_z = |\phi_3\rangle\langle\phi_3| - |\phi_4\rangle\langle\phi_4|, \quad (24)$$

$$J_x = |\phi_3\rangle\langle\phi_4| + |\phi_4\rangle\langle\phi_3|. \quad (25)$$

Then the total Hamiltonian in the single-excitation subspace becomes

$$H_s = \lambda J_z + \sum_j \omega_j b_j^\dagger b_j - J_x \sum_j g_j (b_j + b_j^\dagger) \quad (26)$$

which is a standard spin-boson model [24], with the coupling strength between the “spin” and the bosonic bath being $g_j = \kappa_{1j} - \kappa_{2j}$. Then both the decay and decoherence rates of the “spin” in the energy representation are proportional to $\sum_j (g_j)^2$. This result coincides with the results from the master equation [see Eq. (19)].

In the non-single-excitation subspaces, it satisfies that $\sigma_z^{(1)} - \sigma_z^{(2)} = 0$, and the interaction Hamiltonian becomes

$$\begin{aligned} H_{I,ns}^{\text{com}} &= \sum_j \frac{\kappa_{1j} + \kappa_{2j}}{2} (\sigma_z^{(1)} + \sigma_z^{(2)}) (b_j^\dagger + b_j) \\ &= (|\phi_1\rangle\langle\phi_1| - |\phi_2\rangle\langle\phi_2|) \sum_j (\kappa_{1j} + \kappa_{2j}) (b_j^\dagger + b_j). \end{aligned} \quad (27)$$

where the second subscribe “ns” implies the non-single-excitation subspaces.

Similar to the case in single-excitation subspace, we can define the collective operator

$$\mathcal{J}_z = |\phi_1\rangle\langle\phi_1| - |\phi_2\rangle\langle\phi_2|, \quad (28)$$

then the total Hamiltonian in the non-single-excitation subspaces yields

$$H_{ns} = \Omega \mathcal{J}_z + \sum_j \omega_j b_j^\dagger b_j + \mathcal{J}_x \sum_j \xi_j (b_j^\dagger + b_j) \quad (29)$$

which is a pure dephasing Hamiltonian for a “spin” with the coupling strength between the spin and the bosonic bath being $\xi_j = \kappa_{1j} + \kappa_{2j}$. Therefore, in the energy representation, it experiences only the decoherence but without dissipation and the decoherence rate is $\gamma_\varphi \propto \sum_j (\xi_j)^2$. This fact also coincides with the results from the master equation [see Eq. (15)].

IV. DYNAMICS IN MULTIPLE COUPLED-QUBIT SYSTEM

In Sec. III, we have studied the collective dephasing of two coupled qubits with a common bath. Now, we generalize it to the system of N (> 2) coupled qubits.

For the system of N qubits, the eigen-energies of the Hamiltonian H_S in Eq. (1) in the single-excitation subspace are

$$E_n = \Omega(1 - \frac{N}{2}) + 2\lambda \cos(\frac{n\pi}{N+1}), \quad (30)$$

where $n = 1, 2, \dots, N$ is taken as integers and the corresponding eigen-states are

$$|\psi_n\rangle = \sqrt{\frac{2}{N+1}} \sum_{j=1}^N \sin(\frac{nj\pi}{N+1}) \sigma_+^{(j)} |G\rangle, \quad (31)$$

where $|G\rangle$ represents that all the qubits are in their ground states.

We further write the interaction between the qubits and the common bath as

$$H_I^{\text{com}} = \sum_{\alpha=1}^N Z^{(\alpha)} \otimes R^{(\alpha)}, \quad (32)$$

where $R^{(\alpha)} = \sum_j \kappa_{\alpha,j} (b_j^\dagger + b_j)$. As before, we have written $\sigma_z^{(\alpha)}$ as $Z^{(\alpha)}$ for compactness. The Lindblad master equation under Born-Markov and secular approximations is expressed as [1]

$$\frac{d}{dt} \rho = -i[H_s, \rho] + D[\rho] \quad (33)$$

with

$$\begin{aligned} D[\rho] &= \sum_{\omega, \alpha, \beta} \gamma_{\alpha\beta}(\omega) [\tilde{Z}_\beta(\omega) \rho \tilde{Z}_\alpha^\dagger(\omega) - \frac{1}{2} \tilde{Z}_\alpha^\dagger(\omega) \tilde{Z}_\beta(\omega) \rho \\ &\quad - \frac{1}{2} \rho \tilde{Z}_\alpha^\dagger(\omega) \tilde{Z}_\beta(\omega)], \end{aligned} \quad (34)$$

where

$$\gamma_{\alpha\beta}(\omega) = \begin{cases} \sqrt{J_\alpha(\omega) J_\beta(\omega)}, & \omega \geq 0 \\ 0, & \omega < 0 \end{cases}, \quad (35)$$

and

$$\tilde{Z}_\alpha(\omega) = \sum_{E_n - E_m = \omega} \langle \psi_m | Z^{(\alpha)} | \psi_n \rangle | \psi_m \rangle \langle \psi_n |. \quad (36)$$

In the summation of Eq. (34), the terms with $\beta = \alpha$ come from the dephasing of the individual qubits, which are always being for either the case that the qubits share a common bath or interact with their local baths independently. However, the terms for $\beta \neq \alpha$, which come from the interference of dephasing processes for different qubits, exist only when the qubits share a common bath.

Now, let us take the case of $N = 3$ as an example to show how the interference between the dephasing processes of different qubits induced by the common bath affects the dissipation of the eigen-states of the system. To this end, we prepare the system in the state $|\psi_i\rangle$ ($i = 1, 2$) initially and calculate the probability P_i for the system still in the initial state respectively. By solving the master equation [Eq. (33)] numerically, we plot P_i as functions of the evolution time in Fig. 4. For comparison, we also plot the results when the different qubits interact with their local baths independently. In Fig. 4, we use the notations P_i^{com} and P_i^{ind} to represent the situation with common and independent baths, respectively. It is clearly shown that, the common bath induced interference slows down the dissipation of the system. To study the interference effect in detail, we apply Fermi's golden rule [25, 26] to calculate the decay rates of P_1 and P_2 which are denoted by γ_1 and γ_2 respectively. The results are given by $\gamma_i = \gamma_i^{\text{ind}} + \gamma_i^{\text{int}}$ ($i = 1, 2$) where

$$\gamma_1^{\text{ind}} = \frac{1}{2}J_1(\omega_{12}) + \frac{1}{4}J_1(\omega_{13}) + J_2(\omega_{13}) + \frac{1}{2}J_3(\omega_{12}) + \frac{1}{4}J_3(\omega_{13}), \quad (37a)$$

$$\gamma_2^{\text{ind}} = \frac{1}{2}J_1(\omega_{23}) + \frac{1}{2}J_3(\omega_{23}), \quad (37b)$$

$$\gamma_1^{\text{int}} = -\sqrt{J_1(\omega_{12})J_3(\omega_{12})} - \sqrt{J_1(\omega_{13})J_2(\omega_{13})} + \frac{1}{2}\sqrt{J_1(\omega_{13})J_3(\omega_{13})} - \sqrt{J_2(\omega_{13})J_3(\omega_{13})}, \quad (37c)$$

$$\gamma_2^{\text{int}} = -\sqrt{J_1(\omega_{23})J_3(\omega_{23})}. \quad (37d)$$

In the above equations, the terms γ_i^{ind} are the decay rates when the different qubits couple to their local baths independently, and the terms γ_i^{int} represent the contribution of the interference arising from the common bath. Obviously, the dephasing of each qubit interferes with each other, leading to a slower dissipation of the system in the single-excitation subspace. We have numerically checked that the decay rates given by Fermi's golden rule coincide with the results from the master equation very well.

In Fig. 5, we demonstrate the decay of the excited states more intuitively. Due to the interaction between the bath and the system, the excited state $|\psi_1\rangle$ will decay to the states $|\psi_2\rangle$ and $|\psi_3\rangle$ simultaneously with the decay rates γ_{12} and γ_{13} respectively. Meanwhile, $|\psi_2\rangle$ will decay to $|\psi_3\rangle$ with the decay rate γ_{23} . The decay rates γ_{12} , γ_{13} , and γ_{23} can also be given approximately by Fermi's golden rule for both independent and common bath. Then, the probabilities P_i satisfy the equations

$$\frac{d}{dt}P_1 = -(\gamma_{12} + \gamma_{13})P_1, \quad (38)$$

$$\frac{d}{dt}P_2 = -\gamma_{23}P_2 + \gamma_{12}P_1. \quad (39)$$

From the above equations, we can observe that $\gamma_1 = \gamma_{12} + \gamma_{13}$ and $\gamma_2 = \gamma_{23}$. That is, P_i ($i = 1, 2$) exhibits an exponential decay when the system is initially prepared

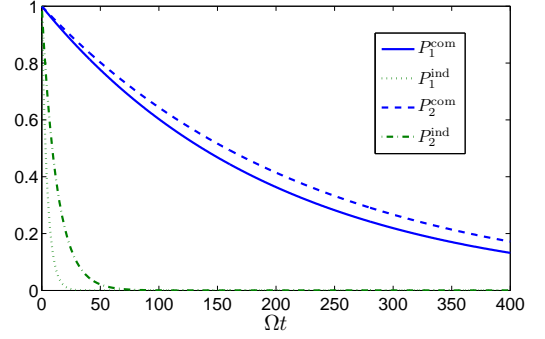


FIG. 4. (Color online) The probability P_1 and P_2 as functions of the evolution time t for the system of three coupled qubits. The coupling strength between the neighbour qubits is set as $\lambda = 0.2$, and the dissipation coefficients are chosen as $\chi_i = 0.1 \sin(i\pi/2N)$ ($i = 1 \cdots N$). All the parameters are in units of $\Omega = 1$.

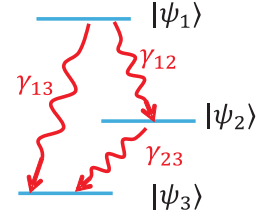


FIG. 5. (Color online) Schematic illustration of decay of the excited states for three coupled-qubit system.

in the state $|\psi_i\rangle$. It agrees with the numerical predictions as shown before.

We can also extend our discussions to the system of $N (> 3)$ coupled qubits. In such a system, we calculate the decay rates of the first excited states in the single-excitation subspace numerically. As shown in Fig. 6, the decay rates are much lower when the different qubits share a common bath (denoted by γ_1^{com} and represented by the red line with solid circles) than those with inde-

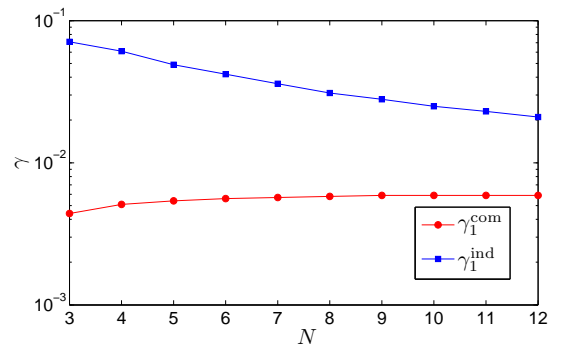


FIG. 6. (Color online) The decay rates of the first excited state in the single-excitation subspace as a function of the number of qubits N . The parameters are the same as those in Fig. 4.

pendent baths (denoted by γ_1^{ind} and represented by the blue line with solid rectangles). Furthermore, the decay rates decrease with the increase of the number of the qubits N when they interact with their local baths independently. In contrast, it tends to be a constant as the increase of N for the case of common bath due to the complicated interference processes among different qubits during the time evolution.

V. REMARKS AND CONCLUSIONS

In experiments, our theoretical model can be realized in superconducting qubits array system. In such kind of systems, the neighbored qubits couple to each other via capabilities and the dephasing of the qubits mainly comes from the $1/f$ noise, which is induced by the charge or flux fluctuations [7].

In conclusion, we have investigated the collective dephasing behavior in the system of coupled qubits interacting with the common and independent baths. In the situation of the common bath, we show that the collective dephasing will induce the dissipation and decoherence of the system in the energy representation. On one

hand, we studied the system of two coupled qubit, which respectively shows the slower dissipation (together with decoherence) in the single-excitation subspace and faster dissipation (but without decoherence) in the non-single-excitation subspaces. The exotic phenomena can be explained from the viewpoint of quantum instructive and constructive interference between the dephasing of the two qubits. On the other hand, we studied the system composed of multiple qubits, and show how the interference effect slows the dissipation of the system in the single-excitation subspace. Both for the system of two and multiple coupled qubits, our results can be understood from the viewpoint of Fermi's golden rule in the energy representation.

ACKNOWLEDGMENTS

We thank the helpful discussions from S. W. Li. This work is supported by NSF of China under Grants Nos.11121403 and 11174027. DLZ is supported by NSF of China under Grant No.11175247, and NKBRF of China under Grants Nos.2012CB922104 and 2014CB921202.

-
- [1] H. P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, Oxford, London, 2002).
 - [2] V. Ramakrishna and H. Rabitz, Phys. Rev. A **54**, 1715 (1996).
 - [3] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, UK, 2000).
 - [4] Y. Makhlin, G. Schön, and A. Shnirman, Rev. Mod. Phys. **73**, 357 (2001).
 - [5] J. Hauss, A. Fedorov, C. Hutter, A. Shnirman, and G. Schön, Phys. Rev. Lett. **100**, 037003 (2010).
 - [6] J. Q. You and F. Nori, Nature (London) **474**, 589 (2011).
 - [7] Z. L. Xiang, S. Ashhab, J. Q. You, and F. Nori, Rev. Mod. Phys. **85**, 623 (2013).
 - [8] Y. Makhlin and A. Shnirman, Phys. Rev. Lett. **92**, 178301 (2004).
 - [9] K. Kakuyanagi, T. Meno, S. Saito, H. Nakano, K. Semba, H. Takayanagi, F. Deppe, and A. Shnirman, Phys. Rev. Lett. **98**, 047004 (2007).
 - [10] S. M. Anton, C. Müller, J. S. Birenbaum, S. R. Oelley, A. D. Fefferman, D. S. Golubev, G. C. Hilton, H. M. Cho, K. D. Irwin, F. C. Wellstood, Gerd Schön, A. Shnirman, and J. Clarke, Phys. Rev. B **85**, 224505 (2012).
 - [11] M. B. Plenio and S. F. Huelga, New J. Phys. **10**, 113019 (2008).
 - [12] M. Mohseni, P. Rebentrost, S. Lloyd, and A. Aspuru-Guzik, J. Chem. Phys. **129**, 174106 (2008).
 - [13] J. Q. You, X. D. Hu, and F. Nori, Phys. Rev. B **72**, 144529 (2005).
 - [14] A. D. Arrigo, A. Mastellone, E. Paladino, and G. Falci, New J. Phys. **11**, 115006 (2008).
 - [15] M. Ban, Eur. Phys. J. D **58**, 415 (2010).
 - [16] Y. H. Ji, Y. X. Yu, and Y. Hu, Adv. Studies Theor. Phys. **5**, 405 (2011).
 - [17] L. D. C. Pulido and R. Aguado, Phys. Rev. B **77**, 155420 (2008).
 - [18] J. P. Santos and F. L. Semiao, Phys. Rev. A **89**, 022128 (2014).
 - [19] Z. H. Wang and D. L. Zhou, Phys. Rev. A **89**, 013809 (2014).
 - [20] J. P. Paz and A. J. Roncaglia, Phys. Rev. Lett. **100**, 220401 (2008).
 - [21] L. Mazzola, S. Maniscalco, J. Piilo, K. A. Suominen, and B. M. Garraway, Phys. Rev. A **79**, 042302 (2009).
 - [22] F. Benatti, R. Floreanini, and U. Marzolino, Phys. Rev. A **81**, 012105 (2010).
 - [23] R. H. Dicke, Phys. Rev. **93**, 99 (1954).
 - [24] A. J. Leggett, S. Chakravarty, A. T. Dorsey, M. P. A. Fisher, A. Garg, and W. Zwerger, Rev. Mod. Phys. **59**, 1 (1987).
 - [25] M. Orszag, *Quantum Optics: Including Noise Reduction, Trapped Ions, Quantum trajectories, and Decoherence*, (Springer-Verlag, Berlin).
 - [26] M. Hellery, Phys. Rev. A **24**, 933 (1981).
 - [27] C. Cohen-Tannouji, J. Dupont-Roc, and G. Grynberg, *Atom-Photon Interactions: Basic Process and Applications* (Wiley & Sons, New York, 1998).
 - [28] P. Zanardi and M. Rasetti, Phys. Rev. Lett. **79**, 3306 (1997).
 - [29] D. A. Lidar, I. L. Chuang, and K. B. Whaley, Phys. Rev. Lett. **81**, 2594 (1998).
 - [30] J. Kempe, D. Bacon, D. A. Lidar, and K. B. Whaley, Phys. Rev. A **63**, 042307 (2001).